Being Of Old Slow To End War: A Game Theory Analysis of the Final Months of 1914 And Why Wars Don't End

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Abstract

The central question of World War I is why a conflict no one wanted lasted beyond 1914. Despite the Kaiser's promise that the guns of August would fall silent before the autumnal leaves fell, the war persisted. The chaotic and controversial July Crisis set the stage, but it was the events of late 1914 and the continuation of hostilities that ensured the conflict would dismantle European hegemony, topple four empires, and lay the groundwork for World War II and the Cold War, cementing the summer of 1914 as Europe's last. By applying game theory—an analytical tool typically used predictively in political science, evolutionary biology, and economics—we explore its descriptive power in detailing the reasons behind the prolonged conflict. Game theory assumes the rationality of the decision-makers of late 1914 to formally describe both their choices and optimal strategies. This application in a systematic model reveals the underlying strategic calculations (and miscalculations) that extended the war well beyond its predicted duration, providing an understanding of the factors that perpetuated the conflict. By interpreting the results through this lens, this study contributes to the larger field of historical conflict analysis, offering insights that are applicable to both historical and modern-day conflicts, as well as testing current models of war termination in a real and well-studied conflict.

> The Lacedæmonians, though fully aware of it, opposed it only for a little while, but remained inactive during most of the period, being of old slow to go to war except under the pressure of necessity, and in the present instance being hampered by wars at home; until the growth of Athenian power could be no longer ignored, and their own confederacy became the object of its encroachments. - Thucydides, 5th Century, BCE[1]

Introduction

In 2019, Shawn Faulkner and Scott Stephenson were asked why, after their lecture on the First Battle of the Marne, the war continued after the devastating losses of late 1914. What could have possibly compelled the belligerents to continue, given their knowledge that a long war would destroy their economies and a generation of men? They argued the sunk cost of those sacrifices compelled both coalitions to continue the war, to make those sacrifices possible.[2]

On the face of it, this seems like an unsatisfactory answer. Had both factions truly fallen for such a trivial logical error? There are many tragedies of World War 1, but if true, this might be the most affecting. A case for forgiveness could be made if the belligerents, seeing the destruction wrought by industrial warfare, had negotiated in late 1914–no one could have imagined that carnage without experiencing it. The case becomes much weaker by the end of 1918.

Instead of committing a logical fallacy, can it be shown that the continuation of the war by both sides was, in fact, rational? The emerging field of game theory might be utilized here to examine the strategic choices both sides face and make sense of their actions. Much work in game theory on war termination in general has already been done. In 1979, for instance, Donald Wittman's model presented a case for rational decision-making, explicitly stated for conflict resolution. James D. Fearon, in 1995, further defined conditions for war termination. In the 21st century, Darren Filson and Suzanne Werner proposed a comprehensive model in a standard format to explain how wars end. Today, political science professor William Spaniel's YouTube channel, which covers the current conflict in Europe with biweekly uploads and a viewership of hundreds of thousands of subscribers,[3] applies these theories to specific and demonstrable case studies. Here, however, we will apply these concepts and principles retroactively on a war with a known end date and, using these tools, determine why it did not end sooner given the popularly held 'irrationality' of the four-year-long conflict.

Overview of Game Theory and its History Concerning World War I

As we know it today, game theory, the math of strategic choices, began over a poker game. John von Neumann, a polymath who accelerated the development of the nuclear bomb, the invention of the computer, and the structure of DNA, was attempting to figure out the most rational response of a poker player with imperfect information.[4] In 1928, he published "On the Theory of Games of Strategy"[5] and became the father of game theory.[6] Game theory has important and wide-reaching predictive power in evolutionary biology, public policy, political science, and economics. It also has been used in

military contexts, which we will cover, but in the discipline of history, it is not as deeply rooted, so a brief overview of concepts will be helpful.

As game theory seeks to quantify decisions, often the utility of the options available to a player (someone or thing who makes decisions and/or follows a strategy) will be given a variable. Often, the actual values of the variables are not important. Game theorists may find important value in the relationship of variables. If one variable going up causes another to go down, or stay the same, that can be interesting and say something about not only the strategy of the players, but the game itself. While single-player game theory exists, the vast majority is between multiple players (and usually specifically two). In games, even zero-sum ones, it is important to remember that players will seek to maximize their own utility, regardless of the score of their opposite (unless their opposite having more utility than themselves in fact gives negative utility, though that dynamic should already be factored into the game). For instance, if there is a game where you have to decide between you receiving $99 and the other player receiving $0, or you receiving $100 and the other player receiving $1,000, you should always choose the second option.

Often, games like these are repeated or iterated. In game theory, players remember past interactions and may adjust their strategies accordingly. However, it is assumed that future interactions matter less than current ones. If you played our hypothetical (and very generous) game every year, the $100 you would get in ten years is worth less than the $100 you get now. You or the other player may die, I may run out of money, the rules themselves may change, the collapse of the dollar might occur, etc., and thus game theory discounts future payoffs with small delta, or δ. δ is a number between 0 and 1 exclusively and represents how much you value the future. If you think the other player is soon to leave the game, or the currency will collapse, you may not value the $100 as much in 10 years, and thus δ will be low. However, if you have high expectations that you will make it to 10 years (both you and the other player are young and healthy, the currency seems stable, etc.), then you would have a high δ value. The calculation is *amount* times δ^t, where *t* is the time period or round in the game. So, if you get $100 now, in 10 years time (10 periods), if you, say, value δ at .5 (that for each period $t + n$, you value the result at only half of $10 \cdot (0.95^{10}) \approx 59.8737$) then you value that money at $\sum_{t=0} \delta t\,(C-I)$, or about a dime. If, however, you set δ at .95 (meaning you value the future a lot), you get $10 \cdot (0.95^{10}) \approx 59.8737$, or $59.87, a much better outcome.

We should also briefly consider World War I's influence within game theory. Robert Axelrod's famous work both in political science and, with W. D. Hamilton, evolutionary biology, culminated in *The Evolution of Cooperation*, featuring an entire section on tit-for-tat, a famous game theory strategy in an iterated game of Prisoner's Dilemma, in the context of the trench system and

the Christmas Truce.[7] Frank C. Zagare in 2011 did something of the reverse of our inquiry and used game theory to explore the beginning of the conflict.[8] The war was even used as the entire premise for teaching the ideas behind game theory by Scott Wolford in his 2019 textbook.[9] For our purposes, however, James D. Fearon's use of the Great War in his seminal work "Rationalist Explanations for War"[10] is the most pertinent. While not primarily focused on the conflict, the paper uses the war in multiple examples, more than any of the other wars mentioned combined. And it is in Fearon's work where we will focus on our last game theory lesson, this time specifically dealing with the central paradox of war and how states internalize costs on actual maps.

Fearon notes that there is a seeming paradox for game theory and the existence of war. He writes,

> As long as both sides suffer some costs for fighting, then war is always inefficient *ex post*–both sides would have been better off if they could have achieved the same final resolution without suffering the costs (or by paying lower costs). This is true even if the costs of fighting are small, or if one or both sides viewed the potential benefits as greater than the costs, since there are still costs. Unless states enjoy the activity of fighting for its own sake, as a consumption good, then war is inefficient *ex post*.[11]

War, in game theory terms, therefore has quite a bit of utility to overcome. Fearon (building on a lot of work we will cover in a moment) contends that this can all be boiled down into proverbial "lines on maps," with disparate variables such as human cost, political will, and future expectations of battle outcomes all being representable by territory gained (or any metric one wishes to use).[12] It is a subtle claim, and not the focus of the paper, but it allows for a more nuanced and wide-ranged analysis, of which we will take full advantage.

Contextual Analysis of Late 1914 and the Promise of a Short War

Before we begin to look at war termination theories, we must quickly consider the context of late 1914. This allows us to apply our upcoming hybrid theory to test the variables surrounding the last time the conflict could be limited to an 1870-71 Franco-Prussian sized war, keeping it from becoming an industrial-powered, worldwide, Napoleonic War. The assumptions of policymakers generally about the war will be examined and a brief position of the pieces on the metaphorical chessboard will outline the backdrop to game theory, rationalizing and quantifying the problem at hand. That the war would be short was believed by nearly every major decision-maker. Kaiser Wilhelm reportedly told his troops leaving for war that they would be home before the leaves fell.[13] There were a few who had inklings of a long war; Fredrick

Engels in 1887 commented,

> And, finally, the only war left for Prussia-Germany to wage will be a world war, a world war, moreover, of an extent and violence hitherto unimagined. Eight to ten million soldiers will be at each other's throats and in the process they will strip Europe barer than a swarm of locusts. The depredations of the Thirty Years' War compressed into three to four years and extended over the entire continent; famine, disease, the universal lapse into barbarism, both of the armies and the people, in the wake of acute misery; irretrievable, dislocation of our artificial system of trade, industry and credit, ending in universal bankruptcy; collapse of the old states and their conventional political wisdom to the point where crowns will roll into the gutters by the dozen, and no one will be around to pick them up…[14]

This is prophetic but is well before the time we are looking at. Sir Edward Grey, Britain's Foreign Secretary, famously noted, "The lamps are going out all over Europe, and we shall not see them lit again in our lifetime,"[15] but this was on the cusp of war, and Grey had cause to be melancholic, as he had tried to keep Albion neutral. Other British voices were much more optimistic. The world of 1914 had been informed by the likes of Norman Angell and his book *The Great Illusion*, which argued that any war would be so disruptive to trade and the economy that it could not be sustainable for any length of time.[16] His counterpart in Russia, Jan Gotlib Bloch (Ivan Stanislavovich Bloch in Russian), also contributed to the idea that the agricultural and manpower devastation (along with similar economic concerns) meant that while military leaders may desire a long war, it simply would not be feasible.[17]

By 1914, in fact, not only was the idea of a long war deemed impossible, but even a short war was considered unlikely. As well, the political climate gave no hint of the coming tempest. As late as May 1914, permanent undersecretary of foreign affairs Arthur Nicholson wrote "Since I have been at the Foreign Office I have not seen such calm waters."[18] The shadow of the future seemed unusually short. After the Battle of the Marne, both sides began to dig into the earth in earnest, forming massive trenches to reduce battlefield casualties and thus reduce cost.[19] This drastically lowered the men necessary to hold a position and thus reduced casualties from the open warfare of the Frontiers and the Marne.[20] The front, at least in the west, was now established, with an expectation of a short war, and, at least by the end of 1914, a much lower rate of casualties than the start of the war. The rate of change of costs was decreasing, and the expectation that those costs would continue was extremely low.

Models for War Termination

A review of the literature surrounding war termination utilizing game theory comes to us from the ashes of World War II when the discipline *in toto* was coalescing. Specifically, this early focus on rationality was in response to the existential threat nuclear war posed. The discipline itself emerged from an earlier quandary of the same type, though somewhat quaint by modern standards of nuclear ordinance: conventional strategic bombing. The phrase "the bombers will always get through"[21] is the cold crucible from when modern nuclear war theory began. Grim calculations, pioneered by physicist and futurist Herman Kahn, brought game theory concepts such as MAD and NUTS to the forefront of American understanding of the new atomic reality (as well as such other eldritch terms like "megadeaths"[22] and "dead hand"). But when nuclear fire did not manifest, a second generation of game theorists began to write about the end of more traditional wars which, though no longer on the scale of the world wars, showed no signs of abating. Much like Clausewitz's concepts of total war and war by algebra, these were largely theoretical and designed to explain trends and forces rather than offer explanations for specific wars. Here, we will consider the history and conclusions of this second generation, arguing that no single formulation can be universally applied, though each provides the tools necessary to construct a new theory for specific wars and our focus, World War I.

Our first model comes at the tail end of the 1970s. Donald Wittman's foundational premise is simple: "[a]n agreement (either explicit or implicit) to end a war cannot be reached unless the agreement makes both sides better off; for each country the expected utility of continuing the war must be less than the expected utility of the settlement."[23] Like much of game theory, Wittman's model relies on utility to each side in regards to continuing or ending a war. Wittman places both on a spectrum, with the unconditional surrender of either side on opposite ends, and plots the expected utility of war termination at time (t) for each side. If the expected utility of war termination is greater than the expected utility of victory times, the internalized estimation of the chance of said victory (P), we can expect them to seek a settlement. If this is also true of the other belligerent, an explicit or implicit peace is likely.

Wittman's work laid much of the foundation for war termination theory. For instance, Wittman points out that changes to P do not necessarily mean a negotiated settlement is more likely. If, for instance, Country Y's chances of victory decrease, this does not correlate to an end to the conflict. This is because as P drops for Y, $(1 - P)$, the chances of victory for Country X, Country Y's opponent, will likely (though not necessarily) increase; so too will the utility required to satiate this updated calculation. What was once

acceptable war termination numbers at t - 1 are no longer, and no peace is expected under the theory.[24] Wittman himself starkly notes "… such phrases as, 'We are bombing them in order to bring them to the negotiating table'; 'all we want and have ever wanted in a negotiated settlement is …'; 'the better we do the more the enemy will come to terms and the sooner we will reach a settlement,' should be taken with a grain of salt, for the better '*we*' do the more we will demand."[25]

Wittman also concludes that a "reduction in the intensity of the fighting"[26] counter-intuitively may lower the chances of a settlement. As the expected costs go down–from fewer battle casualties, less material loss, and less political capital loss from those numbers being reported–the expected utility from continued hostilities at t + 1 actually increases as long as P does not decrease (or (1 – P) for the opponent).[27] This is especially important for the model we shall construct, as the opening months of World War I were especially brutal; by the end of 1914, Allied casualties were estimated at 1.14 million on the Western Front alone,[28] but the move from open fighting to trench warfare, brutal as the latter was, lowered the costs. This is the first puzzle piece game theory sets into place–battlefield changes altered the game such that there was less utility cost in continued fighting. While we are specifically looking at the period right after the Battle of the Marne, before trench warfare could prove its worth in lowering the costs of fighting, this is still an important explanation for why the war lasted as long as it did. In fact, only until 1918 when casualty figures reached what they were in 1914[29] did a settlement become more likely and in fact did happen. When δ and costs were high enough, a settlement was not far behind.

While we touched upon Fearon's ideas earlier, it is relevant to identify his importance here and consider his model more closely. Again, remember Fearon's fundamental paradox concerning war, that it is inefficient and *prima facæ* irrational.[30] In other words, both sides, if they knew the specific settlement of a war, would prefer to jump straight to that settlement, regardless of what it looked like, than receive that settlement *and* pay the costs of war. Yet, of course, wars still occur. If game theory is to be of academic value, it must have a response to this seeming paradox; rational actors would avoid war and accept settlements if they had an accurate assessment of their enemies and their own capabilities (we might even see here echoes of Sun Tzu, who claimed that in knowing the enemy and knowing oneself, one would never be in peril in battle[31]–there would be no battle fought if both sides had worked out the game theory strategy beforehand!).

To solve this paradox, Fearon posits three arguments in response (he cites two more that only partially explain this phenomenon, as well). Firstly, "rational leaders may be unable to locate a mutually preferable negotiated settlement due to *private information* about relative capabilities or resolve and

incentives to misrepresent such information."[32] Put simply, in the calculus about changes to the status quo, actors may have knowledge (usually about their own position) that the other side does not, and, interdependently, have reason to lie about that information. Secondly, commitment problems may make it impossible to ensure neither side reneges on the bargain upholding peace. Finally, an idea that Fearon admits is less compelling is the idea of *issue indivisibility*, that "some issues, by their very natures, simply will not admit compromise."[33] The lessons he draws can not only inform our own formulation, but his influence stretches to all future works in war termination game theory we will consider.

Finally, we turn to Darren Filson and Suzanne Werner's model, which attempts to connect war initiation and termination in a single framework. Their work is far more ambitious than Wittman's, forwarding a proposal-rejection-battle cycle in standard form, including several decision nodes for each failed proposal. Interestingly, Filson and Werner's model is not simultaneous, as the defender nation (*D*) takes its turn after the attacker's (*A*) proposal. As they put it, "[a] *war* consists of an alternating sequence of negotiations and battles."[34] Pulling explicitly from Fearon, Filson and Werner rely on the hidden information of the defender's capabilities. Ultimately, as the war continues, the attacker updates their beliefs about the defender's capabilities and positional strength. The defender, meanwhile, is seeking only to maintain the status quo but has little ability other than outlasting the attacker.

Filson and Werner maintain that their model provides eleven conclusions, perhaps most importantly that short wars favor the attacker and long wars favor both the defender and peace, but for our purposes, it is not sufficient to explain why World War I lasted as long as it did. While Filson and Werner's framework is certainly comprehensive, it seems most explanatory when dealing with symmetrical (which World War I mostly was) and non-coalitional (which World War I certainly was not) wars. Furthermore, the very foundation of the model, that of an attacker making some sort of demand upon the defender, does not fit well over the July Crisis, as most belligerents sought the status quo and the only one which didn't (Austria) specifically sent a proposal purposefully designed to fail and may very well have rejected *any* response by Belgrade.[35]

However, even for a conflict as unorthodox as the Great War, Filson and Werner's model provides excellent resources for a more tailor-made framework. They not only have a much more focused representation of initial resources (*R*),[36] but also a clearly defined relationship between expectations of victory factoring into war costs.[37] Their model represents a clear evolution

from von Neumann's poker game and even Wittman's formulation of game theory on war termination. With this framework, and that of Fearon's and Wittman's, we have the tools necessary to construct a hybrid model for explaining the military and diplomatic situation at the end of 1914.

Our model will draw significantly on the idea of cost, specifically expected cost. The politicians and generals who reluctantly went to war were not completely blind to the potential costs of conflict. Therefore, borrowing from Filson and Werner, we begin with *R*, or a state's resources. In a modern context, we might think of Russia's infamous war chest that was growing leading up to its 2022 invasion. In 1914, it might include the industrial stored capacity of the Great Powers, the British expeditionary force, the Imperial German Army, etc. Each state will begin with this and draw from it to wage war.

Next, we will model the benefits of victory. *V* represents whatever a belligerent is hoping to gain from the conflict, even if that is maintaining the status quo. Modifying that, we will use Wittman's *P*, though we will use more modern conventions and use the Greek letter omega, or ω, but it is the same as Wittman's formulation–some number between 0 and 1, representing the state's internalized estimation of victory. ωV is the benefits of victory, tempered with the uncertainty of that victory. We add this to *R*.

The costs of fighting will be the most complex to model. Firstly, we will assume that war will have at least some military action–declared wars that are resolved before any casualties may still be wars, but unfortunately our model will not represent them well. So, one fight at least will occur, represented by *C* (in our model, this will be accomplished by setting *t* equal to 0, ensuring at least one unit of *C* will be multiplied by 1). We will add to this the expectation of future battles. We don't know how many future battles there will be, so we will utilize the aforementioned discount variable, or small delta (δ). We'll add all those battles together with summation notation, giving us, for cost:

$$\sum_{t=0}^{\infty} \delta^t (C - I)$$

In layman's terms, *C* represents the per-period cost of fighting. We have included industrial capacity to be subtracted from cost with *I*. Think of this as added resources over the course of the war–factories pumping out material, men coming of age to join the front lines, etc. *I* is also a per-period *t* value.

$$R + \omega V - \left(\sum_{t=0}^{\infty} \left(\delta^t (C - I) \right) \right) \geq 0$$

Finally, we set this whole equation to equal or greater than 0. If it is less than 0, we are running a resource deficit, indicating our prediction of a failed war, one which a rational actor would stop at the first opportunity. Combining these together, we get:

$$R+\omega V - \left(\sum_{t=0}^{\infty} {}_{(2024)}^{17}(\delta^t(C-I)\right) \geq 0$$

Keep in mind this calculation will constantly be updating in the minds of policymakers of the belligerents and, as long as it remains true, a rational actor who believes the war can still be waged. This formulation gives us a working model for what the decision-makers of 1914 considered a sustainable conflict. Finally, we can also express the above inequality as a geometric sequence, which simplifies to:

$$10-\sum_{t=0}^{\infty} 0.1^t = 8\frac{8}{9}$$

This is true so long as $0<\delta<1$, or, more conventionally, $|\delta| <1$, which is in fact true in our case, as δ represents a discount value. This formulation will be easier to manipulate algebraically should we desire to do so.

Application of Models to 1914

A larger, "trends and forces" approach to the causes of World War I imply an Allisonian Thucydides' Trap.[38] The rising power of Germany threatened the status quo of Europe and did so in a shockingly quick way. By the stroke of a pen on January 18, 1871, the largest land power in the world declared itself into existence. One could even extend the argument that the rising power of Serbia contested the waning power of Austria. France, England, and Russia, the established powers, wanted to reign in a surging Germany (and Austria a reticent Serbia). Ruling powers, in the face of new, rising powers and changes to the existing political framework, found themselves in a position, or trap, where preemptive war seemed like the only option to maintain the status quo. It is this idea of fighting a war to maintain the status quo where we will first look.[39] What Allison and Thucydides both argued is a form of a commitment problem.

We touched upon commitment problems with Fearon, but a proper definition is called for. Robert Powell states it thusly:

> [t]he crucial issue in commitment problems is that in the anarchy of international politics, states may be unable to commit themselves to following through on an agreement and may also have incentives to renege on it. If these incentives undermine the outcomes that are Pareto-superior to fighting, the states may find themselves in a situation in which at least one of them prefers war to peace.[40]

In other words, agreements have to be self-enforcing, or no state can be expected to maintain it *even if* it is beneficial to them or if they think the opponent might renege. While this is largely used to describe how wars begin, it can be useful to see why they end, or more importantly, might not end. Under this view (which also oddly places the blame for war on the Triple Entente), Britain, France, and Russia did not solve the underlying issue of German ascendancy. While they had the resources to fight, they then were incentivized to continue that fight until they could restore the status quo of a pre-German Europe (a result they would achieve at the end of World War II). In this scenario, Germany, seeking to maintain its upward trajectory, would prefer not to have war (as Fearon notes, war is inefficient, and Germany was better at allocating its resources, hence its ascendancy).

This, however, does not align with the facts. German military plans called for a wide escalation of the war.[41] It was German reluctance to start an international conference to deal with the growing July Crisis.[42] Britain especially was reticent to enter the war–even by early August, before Belgian neutrality had been violated. France could provide the justification–revanchism was strong in the Third Republic to reclaim Alsace and Lorraine. Those claims, and the political humiliation of Belgrade, might be considered the only pre-war victory conditions of any of the major belligerents. But of the Triple Entente, France was the most passive. Russia and Britain took far more active steps to escalate their commitment to enter the war (though, in the case of the U.K., it still did not desire that war).

The only true commitment problem faced by both belligerent coalitions was that of separate peaces. All sides dealt with allies potentially ending their role in the conflict early, a problem exacerbated by the inability of already warring parties to enforce agreements of collective peace. For instance, Great Britain, and to a lesser extent, France,[43] worried greatly over inviting the erstwhile third member of the Central Powers, Italy, to join them. Prime Minister Asquith thought the matter was important enough to grant every Italian demand simply for the promise not to sign a separate peace; as he wrote to King George V, "[t]he importance of bringing Italy without delay

appears to be so great that it was agreed to give a general consent to what she asks and to press Russia to do the same provided the Italians will agree to bring all their forces into the common stock against all our enemies (including Germany) and will bind themselves not to make a separate peace."[44] Germany, too, constantly worried about the Austro-Hungarian Empire in its death throes, to the point that it largely took over control of the Habsburg Empire (as Thomas Otte points out, Austria's declaration of war against Serbia was the last independent political act of a Habsburg, a dynasty which lasted as a head of state of Europe for over six hundred years).[45]

In our focused model, commitment problems would be a manipulation of the *V* variable, modifying the benefits of victory. If the incentives of *V* were significantly larger than the costs (and the chance of victory was not especially low), war continuation should be expected. But commitment problems in this sense do not explain the continuation of war here. War had been avoided before, in other crises. In 1914, there was no special commitment problem that surfaced compared to previous crises.[46] In fact, given the fear of a separate peace, the commitment problem worked in the opposite direction; no country wanted to be caught holding the metaphorical bag and being the last one in their coalition left in the conflict, a situation Britain would find itself in after the fall of France in 1940.

Lower δ Values and the Problem of Belief in a Short War

While nearly every aspect of the Great War is controversial, one which is less so is the idea of a short war. As covered earlier, policymakers on both sides believed the conflict would not (and, in fact, could not) last long. In our model, we might say that the shadow of the future was not long for the politicians and generals of Europe of 1914. This means a change in δ, or the discount factor. How might this affect war calculations?

Let's put in some numbers and test different values of δ. We'll imagine one of the belligerents begins the conflict with 10 resource units and each period of *t* costs 1 unit (for simplicity, we will assume industrial capacity is already factored into the cost, so *C-I* is just *C*). We'll also forgo victory payoffs for now. Thus, we arrive at:

$$10 - \sum_{t=0}^{\infty} \delta^{t}$$

If we expect a short war, our estimation that the chance the war will last at least until $t + 1$ is, say, 10%, (and the subsequent period after that has a 10% of the war continuing to $t + n$), we might put 0.1 as δ, and thus:

$$10-\sum_{t=0}^{\infty} 0.1^t = 8\frac{8}{9}$$

Meaning, if we stalemate, we can expect to exit the war paying only 10/9 units. If, however, we anticipate a long war and expect the war to continue into $t + 1$ with a probability of 95%, we would instead arrive at the horrible outcome of -10, an amount that would ruin us unless the victory payoff was massive and assured. Incidentally, in this scenario, estimating war at $t + 1$ at a 90% probability equals zero–we would just break even (see fig. 1 for the entire distribution).

In other words, the discount factor δ is used to represent how future costs are valued compared to current costs. Generally, if a country believes there is a low probability of the war continuing, it will discount future costs more heavily, resulting in a lower δ. Conversely, if the country expects a prolonged conflict, it will place more importance on future costs, leading to a higher δ. For example, a δ of 0.5 might be used if there is a perceived 50% chance of the war continuing each period, whereas a δ of 0.1 or 0.9 would reflect lower or higher perceived likelihoods, respectively. It should be noted that while there is typically a correlation between the perceived length of the war and the discount factor, this relationship is not strictly causative. There may be cases where a country internalizes future costs differently than expected based on the anticipated duration of the conflict. However, none of these seem to apply to 1914.

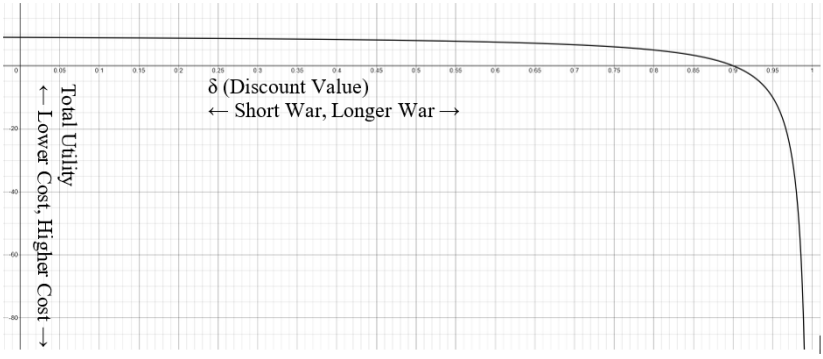

Figure 1: This graph shows the distribution of all values of δ in relation to cost. As the expectation of a long war increase, the expected utility drops precipitously off after a δ of .9.

Image courtesy of Desmos (desmos.com)

This alone presents serious problems for the "war ending early" theory. Ironically, believing that a war will be short can increase its length. Given the predictions of Angell and Bloch we saw earlier, a general sense of a short war, a war that, mathematically, *must* be short, counterintuitively pushed both sides away from the bargaining table–the cost was large, but the huge downward push on δ made those losses seem sustainable. The decision-makers had such a devalued vision of the future in their belief that whatever the outcome, the war couldn't continue. This meant that until the real value of δ was discovered, there was a rational model where continuation, especially with, as we will look at soon, *V* values made to fit an equation solved for *C*. Even for the United States, which in fact could, *ex post,* justify a relatively low δ, the notion that δ must be low had by 1917 and America's entry been shattered. More realistic δ values were used. Winston S. Churchill even mentions in his history of World War II that British plans were finally stretching out to year-long commitments to escalate the war.[47]

Solving For *C*

Another seeming *ex ante* mistake displayed in our model was the *V* values for most belligerents, at least upon their entry into World War I. Only the Austrians and French could point to actual concrete war aims, the humiliation of Belgrade and the recapture of Alsace and Lorraine, respectively. Yet even there, France did not start the conflict to retake their lost 1871 territories, and Austria would argue it was forced to retaliate due to Belgrade provocation. So, our belligerents' *V* values were not simply low, but largely undefined.

The known variables were thus each nations own *R* and *I* values and were furthermore largely fixed (*R* especially so and *I* only changeable slowly, and at great expense taking away from the war effort). The last variable, *C*, was shockingly found out in the period we are looking at, so it is natural to assume the political and military leadership began informally solving our model for *C*. Using our geometric sequence formulation from earlier, we arrive at:

$$R-\delta R+\omega V-\delta\omega V+I\geq C$$

Here is another key to understanding the crisis facing the political leadership of the belligerents. Unable to meaningfully change *I*, *R,* or the new, terrifying *C*, and with ω largely in the hands of their military, politicians and leaders could only consider *V* and δ.

With rising *C* values, a higher *V* or a lower δ could keep the equation in balance. While this did indeed lead to higher *V* expectations (to the point that when the Bolsheviks took over in Russia and published Allied war aims, it was a massive political blow to their leadership, especially to neutrals, for the audacity of it–we can now better speculate as to why *V* had to be so high), we again see a desperate attempt to internalize δ as being low. But as we showed earlier, artificially low δ led to *longer* wars and served as an impediment for peace. Attempting to solve the equation for *C*, trying to "balance the books" given the rising costs, meant leaders were hoping, *and acting as if it were true*, that δ was low. Again, recalling Churchill's memories of Britain's plans ramping up had the war continued into 1919, the full breadth and depth of the war plans laid bare the actual value of *C*, to the point where any reasonable value of *V* Britain could expect wouldn't exceed it, thus London was willing to seek peace.

Sunk Cost and the (Ir)Rationality of Basing Current Decisions on Past Payouts

Let us return to Faulkner and Stephenson's response to why the war didn't end after the Marne. Did sunk cost affect the decision-making of policymakers in late 1914? And how might game theory explain it if it did? The sacrifice of so many lives, as Faulkner and Stephenson put it, placed those men in a dire bind–admitting defeat, or even a white peace would be in effect saying to their populations "all those empty seats at the dinner table were sacrificed for naught." In a democratic system, this could very easily end a political career. An authoritarian ruler, like Wilhelm II, might have a better chance at political survival, but would have a much longer fall should those chances run against him (see his cousin Czar Nicholas II in 1917-1918 or Muammar Gaddafi in 2011).

On the face of it, as Corina Haita-Falah opens with in her work dealing with the subject, "[In]formative economic theory indicates that costs incurred in the past are irrelevant for future marginal payoffs, i.e., sunk costs must be ignored."[48] Since the costs have already been paid and cannot be refunded, they should rationally be ignored in standard game theory. What happened at $t - 1$ has already occurred and mathematically has no place in the current decision beyond what it has already affected. In our example, the losses at the Frontiers and the Marne have already been lost–their roles have been played, and their credits or debits to the war calculation have already been tabulated.

However, as Haita-Falah describes throughout her paper, even in perfect information models where no variables are hidden or guessed at, people still tend to take previous or sunk costs into account.[49] She concludes that as the value of the sunk costs rise, so too does the behavior of players to value their previous investment more and exhibit a greater tendency to act

upon that evaluation (to the point, she describes, "even in the absence of the psychological mechanisms that typically explain it."[50])If this is evident in the low-stakes games played in a university setting, in an unprecedented war of theretofore unimaginable death, it should not be surprising to see Faulkner and Stephenson's point given credence.

Strictly, sunk costs should be ignored, yet if game theory is going to describe the rational actions of people, it must account for this *ex ante* mistake. Fortunately for game theory, we hinted at a possible solution earlier. The answer lies in reformulating the model and incorporating what it actually represents. Policy makers recognize some variable those previous losses signify to their current decision which is not being modeled (or only implicitly so). To account for what that sunk cost represents, we must determine *why* it mattered to the political landscape of 1914 and what kept the belligerents fighting not only despite those casualties, but because of them.
Firstly, as pointed out by Max H. Bazerman, Toni Giuliano, and Alan Appelman, when the responsibility of those sunk costs fall on few shoulders, i.e., when there are few "high responsibility" decision makers for the potential sunk cost, and that situation is being evaluated by a larger group, the necessity to "double-down" on the original decision heightens.[51] This information can be coupled with Filson and Werner's framework modeling the updated values of chances given previous information (in Filson and Werner's case, the number of successful defenses from the attacker's battles updates the attacker's belief in the likelihood of the defender not acquiescing to the attacker's proposal to the change in status quo).[52]

While modeling the specifics of this dynamic is beyond the capabilities of our model, as it would involve adding complexity beyond the scope of this paper,[53] we can still outline the nature of such an addition. Faulkner and Stephenson's answer strikes directly at the home front response to a late 1914 white peace, that it would be politically untenable. This can be modeled; there is some political cost, which we will call L, of not ending the war victoriously, paid specifically by the decision-makers and political leadership. L is slightly different than our earlier variables–the general C we have been working with is paid, rather abstractly, by the entire state. We might further generalize it (as Wittman does when he describes his model in terms of political leadership paying the costs, but generally unconcerned with the dynamics between leadership and population or different leadership factions within one belligerent[54]) as being a military cost. But L is strictly between the political leadership and the ruled class. As long as L is greater than our original model, victory may be unaffected, but white peace may be untenable. Furthermore, L is especially sensitive to $t - n$ costs in a way our

original model is not, which would allow game theory to model a version of sunk costs that the Triple Entente and Central Powers faced in late 1914.

Conclusion

Game theory is a powerful tool in predicting the future but is also useful in examining the mysteries of the past. As we have seen here, there are rational reasons why World War I continued past the catastrophic losses of the Frontiers and the Marne. They elucidated previous work in game theory and considered both players' cost calculations. Firstly, the reduction of intensity lowered the cost of war, contributing to its length. Secondly, the belief itself in a short war narrowed the shadow of the future, further driving down internalized costs. Finally, the manipulation of the victory conditions solved for costs, especially given the relative mutability of territorial acquisition to finely control for costs.

Future research into more complex war termination modeling by including both current frameworks and tailor-made ones for World War I (or indeed any war) is the next logical step for this line of research. A dynamic game with iterated steps and finding a true Nash equilibrium for war termination, tailored to the period in question, would further game theory within the military history field and provide rationalistic explanations for the decision-making of past conflicts. I do not believe that some "grand unifying theory" of war termination is out there, waiting to be discovered–the breadth and depth of the human experience of war contains too many variables to accurately model beyond any one aspect of a single conflict, but by drawing these general lessons and working towards more modeling of specific phenomena, we can begin to build a repository of frameworks to understand how wars end and how we might expedite that process.

Part of a letter to Maude May, June 18th, 1916

I must not allow myself to dwell on the personal – there is no room for it here. Also it is demoralising. But I do not want to die. Not that I mind for myself. If it be that I am to go, I am ready. But the thought that I may never see you or our darling baby again turns my bowels to water. I cannot think of it with even the semblance of equanimity.

My one consolation is the happiness that has been ours. Also my conscience is clear that I have always tried to make life a joy for you. I know at least that if I go you will not want. That is something. But it is the thought that we may be cut off from each other which is so terrible and that our Babe may grow up without my knowing her and without her knowing me. It is difficult to face. And I know your life without me would be a dull blank. Yet you must never let it become wholly so. For to you will be left the greatest charge in all the world; the upbringing of our baby. God bless that child, she is the hope of life to me.

My darling, au revoir. It may well be that you will only have to read these lines as ones of passing interest. On the other hand, they may well be my last message to you. If they are, know through all your life that I loved you and baby with all my heart and soul, that you two sweet things were just all the world to me. I pray God I may do my duty, for I know, whatever that may entail, you would not have it otherwise.

- Captain Charles May (July 27, 1889 – July 1, 1916), 'B' Company, 22nd Batt., Manchester Regiment[55]

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