Richardson's Prediction

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Every day, people rely on accurate, up-to-date weather forecasts to help them plan their days and weeks. Meteorologists are counted on to inform their viewers on whether they will need to dodge rain showers during their Sunday brunch, bring a heavier coat to work in response to colder temperatures, or when they will need to postpone football games or other activities and stay sheltered while a severe thunderstorm passes through their area. While the viewers receive the forecast in simplified language on television, they often don't get exposed to the mathematics that is involved behind the scenes in making weather forecasts. Without mathematics, these forecasts could not be made with much certainty. Despite its essentialness, the use of mathematics in forecasting the weather is a relatively young and overlooked concept. The first successful mathematics applications in the field of weather forecasting would be made in 1922, by a man named Lewis Fry Richardson. By utilizing complex mathematical concepts, namely partial derivatives, Richardson would break barriers in, among other fields, the creation of short- and long-term weather forecasts, providing a template for numerical weather prediction that meteorologists in the coming decades would be able to make stark use of.

Richardson was born in Newcastle, England on October 11, 1881 in a Quaker family. He attended Bootham boarding school in York, England at age 12, receiving a science education with an emphasis in natural history. He attended Durham College of Science in 1898 for math, physics and chemistry; he then moved to King's College in Cambridge, England to study physics, graduating first in his class in 1903 [**8**]. He held off on graduate school until later in his life, receiving a doctorate in Mathematical Psychology in 1947 from the University of London. Richardson had constantly changing interests in the fields of natural science [**8**]. This led him to work at a variety of scientific institutions during his career, including The National Physical Laboratory in the United Kingdom and The Meteorological Office—the UK's equivalent of the United States National Weather Service. His pacifist beliefs passed down to him from his family allowed him to avoid military service during World War I. Richardson's son, Stephen, wrote about his father's pacifistic character in a personal biography [**8**]. According to Stephen, Richardson's unwillingness to participate in World War I would bite him in the future, not allowing him to receive tenure nor a high academic position at any university [**8**]. Richardson died on September 30, 1953 at the age of 71.

Prior to the 1920's, mathematical weather forecasting was still a fairly unprecedented concept. Efforts to predict the weather have been recorded as early as the days of the ancient Greeks, Chinese, and Babylonians. For many centuries, the art of weather forecasting was basically to observe the sky and make a prediction based on what the forecaster saw and what conditions he or she knew to have taken place in that area previously. According to an archive from NASA's Earth Observatory, the Babylonians tried to predict short-term weather changes by observing the appearances of the clouds and the presence of other visible phenomena [**14**]. In ancient Greece, the philosopher Aristotle attempted to describe the properties of various weather phenomena, including lightning, hurricanes, and tornadoes. According to NASA, his writings were considered to be the standard for weather prediction by the general society for nearly two millennia [**14**]. His ideas were largely discarded in the 1600's when it was found that many of his findings about the formation of the various weather phenomena were severely inaccurate [**14**]. While such methods for weather prediction seem rather simplistic today, they were the best means available to civilizations for weather prediction in earlier centuries. It was only with new innovations and research that better techniques would develop and better understandings of weather would become prominent.

The next real innovations in weather forecasting came throughout the late 1800's, as scientists came up with a way to illustrate weather conditions and changes: surface maps. An example of an early surface map is given in Figure 1.

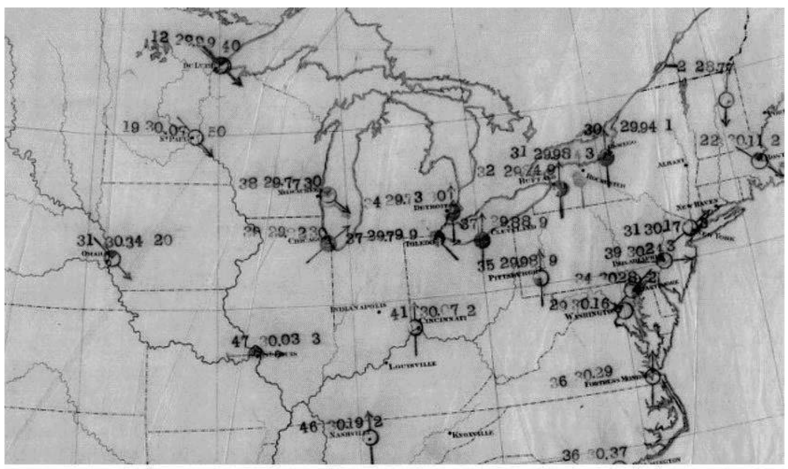

Figure 1. Surface weather map created in 1871 by the National Weather Service [**6**].

Despite this innovation, there still wasn't a well-established, effective means of predicting the weather. Scientists could illustrate weather conditions and future changes on a map. But making accurate predictions for short-term and long-term weather conditions to draw on those maps was still an unprecedented task. Therefore, starting in the 1900's, the attention turned to mathematics as a viable option for forecasting weather. Richardson himself, writing in his book *Weather Prediction by Numerical Process*, did state that he was not the first person to attempt a weather forecast via mathematics [**15**, p. 43]. According to Richardson, an attempt at mathematical forecasting was made in 1908 by Felix Exner [**15**, p. 43]. However, Exner derived a prediction equation using observed temperatures and mean zonal wind calculations to showcase the advancement of a surface pressure pattern, which would not turn out to be of much practical use in making an actual forecast. Therefore, with no real established weather prediction model up to that point, Richardson was delving into new territory with his use of mathematics in forecasting the weather. And in 1922, Richardson would become the first known person to find success in applying mathematics to predicting the weather.

Richardson's first series of weather-related mathematical computations involved changes in pressure at a given location over a 6-hour period that ended in little success. However, the equation most associated with Richardson became known as the Richardson number, named after Richardson himself [**7**]. The Richardson number is a value that Richardson derived to show the relationship between buoyancy and wind shear in the atmosphere. Buoyancy refers to the upward force applied by a fluid on an object based on the density difference between the object and the air around it [**1**, p. 150]. Wind shear refers to the rate of changing of wind speed and direction with height [**1**, p. 233]. The Richardson number equation, in its most basic variation, is written as follows:

$$Ri = \frac{Buoyancy}{Wind\ Shear} = \frac{g}{p}\frac{{}^{dp}/_{dz}}{(\frac{du}{dz})^2}$$

where p equals density, g equals gravity, z represents depth and u represents wind speed [**7**].

The Richardson number is critical in weather forecasting, as it provides a mathematical means of depicting the density of the air in the various levels of the atmosphere, along with the wind speeds in those layers as well. Tornadoes, for instance, are one phenomenon that rely heavily on buoyancy and wind shear for their formation. In fact, according to the National Weather Service, buoyancy and wind shear, along with instability, are the primary ingredients that are measured by forecasters to predict tornado activity [**13**].

Richardson's mathematical techniques heavily involved the use of derivatives, especially partial derivatives. A regular derivative of a function shows a rate of change of a variable with respect to another variable. Partial derivatives, in contrast, are used in functions of more than one variable, only taking the derivative of one variable while holding the others constant [**9**, p. 633]. The Richardson number equations shown above is an example of an equation in which the rate of change in one variable can be shown by taking its derivative while leaving the other variables constant.

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A sample problem demonstrating the use of partial derivatives will follow, but it is first important to understand why a partial derivative is an effective mathematical concept to use for predicting weather. Consider the variable of temperature, for instance. It would be fairly simple to determine the temperature at a given location by just reading a thermometer. If it reads 47 °F, then that's the temperature at that given time. However, that information won't tell how the temperature in that location will change one hour from now, one day from now, or one week from now. To aid in this prediction, consider a function for temperature:

Let the temperature T be dependent on a couple of variables a and b. The function for this setup is

$$T = f(a, b)$$

The partial derivative of the function with respect to a is

$$f_a(a, b)\ ,$$

and the partial derivative of the function with respect to b is

$$f_b(a, b)\ .$$

Suppose that $f(a, b) = ab^3$ and that we want to solve for $f_b(a, b)$. To do this, just take the derivative with respect to b and leaving a constant.

$$f_b(a, b) = 3ab^2$$

Taking the partial derivatives of such functions allows meteorologists to evaluate the rate of changes in certain meteorological variables with respect to other variables, such as the rate of change in temperature in relation to humidity or change in wind speed with atmospheric height, thus allowing them to see patterns or trends and ultimately predict how much or in what ways the weather will change for a given location on a given day or time. Richardson's number is just another one of those functions.

Now, the Richardson number formula is just one equation used in making a single weather forecast, and it's not one that can be easily solved in a short period of time. As demonstrated in the sample problem above, many variables are considered when determining the change in various weather phenomena. Most of the calculations that are used routinely by government weather forecasters, private sector forecasters, and even television meteorologists, would take days, weeks, or even years to perform by hand. For the first twenty years following his meteorological discoveries, Richardson, and weather forecasters like him, would have to perform these tedious mathematical calculations by hand. Even when making his first computations for pressure, according to NOAA, Richardson took six weeks to complete his calculations and only had the help of a slide ruler and a table of logarithms [**11**]. However, starting in the 1940's, computers would come along and begin to save many scientists a lot of time.

In 1946, the first digital programmable computer was introduced. Constructed at the Moore School of Engineering at the University of Pennsylvania, the Electronical Numerical Integrator and Computer (ENIAC) pioneered technology that would be able to perform mathematical calculations too tedious and time-consuming for humans to perform on their own [**3**]. In the coming years, more computers like the ENIAC, and better, would be more widely distributed and available for use at universities and at government weather offices.

In the 1960's, supercomputers would hit the scene, making even faster and more complex mathematical calculations possible. These supercomputers made their way into the National Weather Service offices, allowing the NWS to make the short-term and long-range weather forecasts that many people rely on to this day. According to the National Weather Service page on supercomputers, these supercomputers take data that is collected from radar, weather balloons, satellites, and other technology and perform the mathematical calculations that allow humans to predict every type of weather hazard, including severe weather, extreme temperatures, and even space weather [**12**]. In addition, according to the National Weather Service, the data that is collected is distributed across the United States and around the world, assisting forecasters in other countries in making their own respective weather forecasts [**12**].

The addition of computers and supercomputers clearly saved scientists a lot of time and hard mathematical work that otherwise would have taken years to perform. While Richardson did not live long enough to experience the impacts that computers had on weather forecasts, he was made aware of the advancements made by the ENIAC and eventually supercomputers. Peter Lynch wrote about Richardson's response to the release of computers in his book, *The Emergence of Numerical Weather Prediction: Richardson's Dream* [**5**]. According to Lynch, Richardson responded by saying that, despite the initial batch of calculations produced by the ENIAC taking almost a day to create, the computer was a major advancement in the field of science [**5**, p. 290].

Richardson's research in mathematics, as it applies to weather forecasting, is arguably what he is best known for. However, Richardson used similar mathematical techniques to make contributions in other areas outside of weather. His pacifistic beliefs led him to not want to be involved in any wars. He did, however, find much interest in researching the causes of war and ways to prevent them. It's believed that he was able to find success in applying similar mathematical techniques to these areas as he was able to in the field of meteorology. However, few records of his mathematical success in this field are known to still exist. Thomas Körner wrote about Richardson's life in his book, *The Pleasures of Counting* [**4**]. According to Körner, Richardson had found out that his findings could be of value to chemical weapons designers in creating weapons to help further the war causes, and he did not want to contribute to those efforts [**4**, Chapter 9]. In response, according to Körner, Richardson seized all his research efforts in the field of war and had his findings destroyed before any of them could be made public [**4**, Chapter 9]. This likely explains why hardly any records of Richardson's mathematical contributions to causes and prevention of war are known to exist.

In addition, Richardson was credited for developing his own method for solving a system of equations. The method he came up with is known as a type of iterative method, a method in which successive estimations are used to draw closer to a solution for a system of equations [**2**]. The basic concept for the Richardson iteration method is expressed as follows:

If given a linear equation, written as

$$Ax = b,$$

then the equation in Richardson iteration is written as

$$x^{(k+1)} = x^{(k)} + c(b - Ax^{(k)}),$$

where c is a scalar of the equation such that $x^{(k)}$ is convergent.

Being an iterative method, the idea is that the person doing the calculations will repeat their process, using their estimation again and again to come closer to an approximate solution to the system of equations or, alternatively, determine that the system does not converge. According to David Strong writing for the Mathematical Association of America, an iterative method such as this one is likely the only means of finding a solution for non-linear systems, although it's highly effective for finding the solutions to linear systems as well—particularly equations with several variables that would take a long time to solve by more common methods such as the elimination or substitution methods [**10**].

While credited with mathematical contributions in various fields, discoveries by Richardson, such as the modified Richardson iteration or the war research, are mostly explored only by the most enthusiastic math hobbyists. Richardson is most widely known to modern-day audiences for his contributions in the field of weather forecasting, and his mathematical techniques and expertise are still applied by meteorologists in numerical weather prediction to this day. Richardson's equations are some of the many series of equations that are programmed into the computers and supercomputers used by meteorologists today. According to NOAA, more than 200 million observations are taken each day and processed into regional and global models, which then generates forecasts of various weather phenomena (hurricanes, tornadoes, volcanic eruptions, etc.) as the final products [**11**]. The result of this processing power and access to data is an average computational speed of 14 trillion calculations per second, nearly 15 million model runs generated each day, and almost 6 million global model runs [**11**]. An example of a 48-hour United States model run is given in Figure 2.

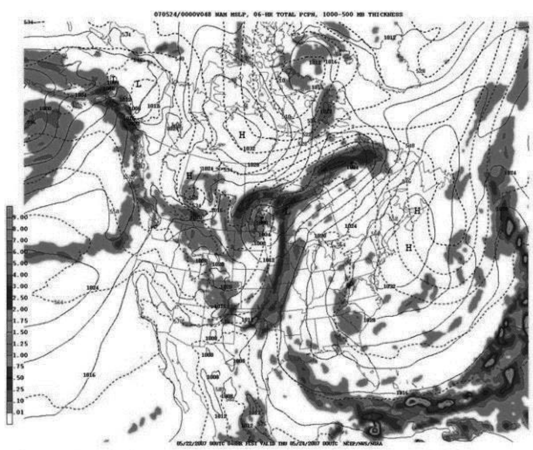

Figure 2. A 2-day forecast run from the North American Mesoscale model showcasing average sea level pressure (in millibars), temperature of the atmosphere and total precipitation (in inches) over a 6-hour period [**11**].

With daily, weekly, monthly, and even yearly weather forecasts becoming more accessible, albeit being accurate no more than 7 days in advance as of this writing, the next major focus for scientists is to create accurate regional and global weather and climate models that can predict the weather conditions a year or more in advance and predict the state of the Earth and its various climates over the periods of years, decades, or even centuries.

The study of meteorology is still a relatively young science. The majority of scientific developments and advancements in the field of meteorology have only come about within the past century. Richardson broke ground in 1922 with his application of differential equations to predicting the weather. He was not the first person to attempt using mathematics towards weather prediction. But he was the first known person to find success in doing so and the first to make such contributions that would prove useful to future meteorologists. With the addition of supercomputers, calculations that took Richardson months to perform can now be done in a matter of milliseconds and several of them at a time. As important as Richardson was in contributing to the study of meteorology, he is not well-renowned outside of the weather community. In the mathematics community, he is nowhere near as recognized as such established great figures as Archimedes or Leonhard Euler. This is understandable, as Richardson was not known to have created a new mathematical theorem or have really made contributions to the well-established fields of mathematics, such as number theory or calculus, outside of his method for solving a system of equations. Richardson is also overlooked likely in part to the fact that much of his work is no longer known to exist and thus can't be analyzed or studied. However, Richardson still deserves a mention when discussing the greatest mathematicians, as his mathematical contributions continue to be used by meteorologists to this day in their efforts to help the general public plan their days. Richardson may be a lower profile, if not underrated, mathematician, in retrospect. But his research and scientific endeavors resulted in mathematical equations that have made it possible to create accurate short-term and long-term weather forecasts that nearly all people rely on. Whether those people realize it or not, it was Richardson who made the art of weather prediction possible.

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