Life and Works of Johann Bernoulli

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Johann Bernoulli was born to Nicolaus and Margaretha Bernoulli on July 27th, 1667, in Basel, Switzerland. Johann was their tenth child and was one of the five children that lived to adulthood (see [**2**, pg 5]). Johann's father pushed him to follow in his own footsteps in hopes that Johann would take over his spice business. However, at the age of 15, Johann knew this was not the career he wanted to pursue. Johann convinced his father to let him study medicine and began his studies at the University of Basel in 1683. Johann's father allowed him to study medicine thinking that Johann would still follow in his footsteps in the spice business (see, e.g., [**7**]).

While Johann studied medicine, he favored mathematics and would study alongside his older brother Jakob Bernoulli. Jakob, like Johann, was pushed by his father to study subjects he despised, such as philosophy and theology. Despite this, Jakob still managed to include mathematics and astronomy in his studies and would become an important mathematician (see e.g., [**4**]). When Johann joined the university, his brother Jakob was already lecturing on experimental physics. Even though Jakob was twelve years older than Johann, it only took Johann two years at the university to become equal to his older brother on the subject of mathematics. The brothers quickly became partners, studying and working on topics together. In the beginning of their mathematical journey, Johann and Jakob seemed to have a close brotherly relationship, as they even lived together alongside Paul Euler (father of Leonhard Euler) while Paul Euler and Johann were undergraduates at the University of Basel (see e.g., [**10**]).

Johann and Jakob became one of the first mathematicians to understand and apply Leibniz's differential calculus. While studying Gottfried Wilhelm Leibniz's theories, Johann would successfully form a close correspondence with Leibniz. This would become the most important correspondence for Johann (see, e.g., [**2** pg. 8]). Alone, Johann would go on to produce many papers on mathematical topics, even whilst working towards his doctoral dissertation in medicine. These papers would include significant results that were encompassed in his correspondence with Leibniz (see, e.g., [**7**]).

After graduating from Basel University, Johann Bernoulli began lecturing on differential equations in Geneva in 1691. From Geneva, Johann would travel to Paris, where he would be introduced to the Malebranche's circle--a leading mathematics and science group in France built by Nicolas Malebranche. Here, Johann would meet many notable mathematicians, one of them being Guillaume de l'Hôpital. This was the start of Johann's engagement with l'Hôpital. This engagement would last many years and involve intense mathematics. l'Hôpital was mainly interested in Johann's knowledge of Lebneiz's newly published calculus methods. This prompted l'Hôpital to request that Johann teach him his methods. Johann agreed on one condition: that l'Hôpital would pay lavishly for his teachings. l'Hôpital was delighted, as there were not many other mathematicians who could offer this great opportunity to lecture on newly discovered mathematics. These lessons would take place in Paris and at l'Hôpital's country home in Oucques. Even when Johann returned to Basel, he was still receiving a great sum of money from l'Hôpital by continuing their correspondence through letters (see, e.g., [**10**]**)**.

l'Hôpital would use the knowledge he learned from Johann's lectures in his own work, which would forever give l'Hôpital a place in the history of mathematics. l'Hôpital published the first calculus textbook *Analyse des infiniment petits pour l'intelligence des lignes courbes* (Analysis of the infinitely small to understand curves) in 1696 that only included a small statement in the preface acknowledging Bernoulli as in [**10**]

> *And then I am obliged to the gentlemen Bernoulli for their many bright ideas; particularly to the younger Mr. Bernoulli who is now a professor in Groningen*.

This book would introduce for the first time, the well-known, l'Hôpital's Rule. The theorem of this rule says:

> Let f and g be functions that are differentiable on an open interval (a,b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a,b), except possibly c itself. If the limit of f(x)/g(x) as x approaches c produces the indeterminate form 0/0, then
>
> $$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$
>
> Provided the limit on the right exists (or is infinite). This result also applies when the limit of f(x)/g(x) as x approaches c produces any one of the indeterminate forms ∞/∞, $(-\infty)/\infty$, $\infty/(-\infty)$, or $(-\infty)/(-\infty)$
>
> Proof:

Consider the case $\lim_{x\to c^+} f(x) = 0$ and $\lim_{x\to c^+} g(x) = 0$.

Define:

$$F(x) = \begin{cases} f(x), & x \neq c \\ 0 & x = c \end{cases} \quad \text{and} \quad G(x) = \begin{cases} g(x), & x \neq c \\ 0 & x = c \end{cases}$$

.For any x, $c < x < b$, F and G are differentiable on (c,x] and continuous on [c,x]. From the Extended Mean Value Theorem, there exists a point z in (c,x) such that

$$\frac{F'(z)}{G'(z)} = \frac{F(x) - F(c)}{G(x) - G(c)} = \frac{F(x)}{G(x)} = \frac{f'(z)}{g'(z)} = \frac{f(x)}{g(x)}$$

Letting $x \to c^+$, you have $z \to c^+$ because $c < z < x$, and

$$\lim_{x\to c^+} \frac{f(x)}{g(x)} = \lim_{x\to c^+} \frac{f'(z)}{g'(z)} = \lim_{z\to c+} \frac{f'(z)}{g'(z)} = \lim_{x\to c^+} \frac{f'(x)}{g'(x)}$$

see [**5**]. This rule is particularly important for mathematicians because it helps us evaluate indeterminate limits of the form 0/0 or ∞/∞.

Johann was reasonably vexed that l'Hôpital would take credit from his own lectures and would later go on to make claims that he was the true author of l'Hôpital's book after l'Hôpital's death in 1704, likely to avoid breaking agreements that had been set between the mathematicians. Johann would not be recognized or believed to be the author until 1922 when evidence was found that Johann had indeed written the proofs for l'Hôpital's book. It has been proved, though, that l'Hôpital corrected some of Johann's mistakes, such as his belief that the integral of $1/x$ is finite (see, e.g., [**5**]). Although it was upsetting for Johann's work to be "stolen," he would later, ironically, and hypocritically, accomplish similar schemes.

Johann's schemes first started with his older brother, Jakob. As noted previously, Johann and Jakob studied together at the University of Basel. For a time, the brothers made an unstoppable genius duo in the mathematics world. However, due to Johann's fiery and jealous nature, their bond soon broke. Their work became competitive and hostile, as both worked on problems to prove who was the better mathematician. In one instance in 1691, the brothers became intrigued by the shape of a ship's sail and raced against each other to be the first one to solve the problem. Jakob had written in a letter to Johann exclaiming that he had solved the problem, but did not explain his solution. Johann immediately solved the problem, too, and published his work before Jakob did, belittling his older brother by publicly announcing that Jakob had apparently given up, despite the fact Jakob had already solved the problem. This would only be the beginning of the quarrel between the brothers, as well as the many other quarrels for Johann (see, e.g., [**11**]).

The conflict and competition between the brothers were petty and childish, as they were rooted in getting the most recognition. However, it must be noted that one could consider their conflict and competition extremely beneficial to other mathematicians, as both brothers made very important contributions to mathematics that were motivated mainly out of spite. Johann and Jakob Bernoulli's final break in their relationship would happen after Johann's brachistochrone challenge in 1696. Johann challenged mathematicians around the world to solve the problem with the question, as seen in [**8**]:

> *Given two points A and B in a vertical plane, what is the curve traced out by a point acted on only by gravity, which starts at A and reaches B in the shortest time.*

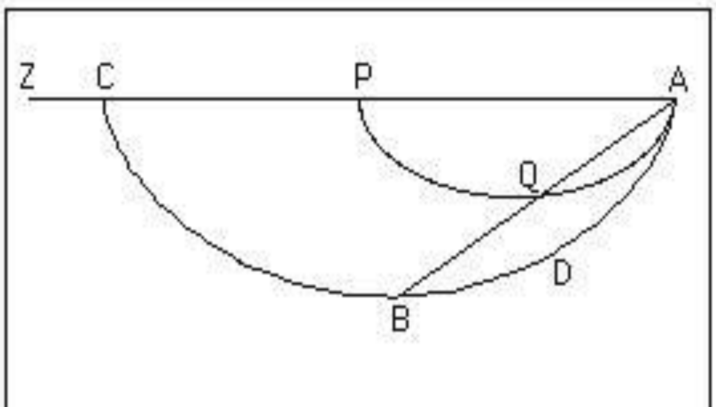
This problem was related to Pascal's challenge regarding the cycloid, of which Johann knew would bring forth the solution to the brachistochrone using the methods of Fermat. The brachistochrone had been attempted before Johann's challenge, such as by Galileo in 1638. However, Galileo had made an error in his work by deducing that the quickest path of descent from A to B would be the arc of a circle, which is false (see, e.g., [**8**]). Johann, along with Leibniz, would intentionally entice their rival, Isaac Newton, which can clearly be seen in his boastful announcement of the challenge [**8**],

> *...there are fewer who are likely to solve our excellent problems, aye, fewer even among the very mathematicians who boast that [they]...have wonderfully extended its bounds by means of the golden theorems which (they thought) were known to no one, but which in fact had long previously been published by others*

The pokes toward Newton in Johann's challenge were successful, and Newton took the bait. Newton's niece, Catherine Conduitt, whom he had been living with, told the story in [**3**]

When the problem in 1697 was sent by Bernoulli - Sir I.N. was in the midst of the hurry of the great recoinage and did not come home till four from the Tower very much tired, but did not sleep till he had solved it which was by four in the mourning

Newton felt had he ignored the brachistochrone challenge, his reputation and honor would be put on the line. This is a reasonable fear for Newton, as it would have been inevitable that both Johann and Leibniz would have mocked his mathematical abilities had he avoided it. It seems that Newton's motivation was fueled by spite, as to make a mockery out of Bernoulli's extended challenge. Newton was able to solve the brachistochrone problem in a matter of hours, while it would have taken other notable mathematicians months. It is reported that Newton said, "I do not love to be dunned and teased by foreigners about mathematical things…" (see [**8**]). Newton sent in his work unsigned and anonymously; however, with an English post and a solution that bore undoubtedly signs of utmost genius, it would be clear that it came from none other than him.



Newton's diagram

Newton's Solution: From the given point A let there be drawn an unlimited straight line APCZ parallel to the horizontal, and on it let there be described an arbitrary cycloid AQP meeting the straight line AB (*assumed drawn and produced if necessary*) *in the point Q, and further a second cycloid ADC whose base and height are to the base and height of the former as AB is to AQ respectively. This last cycloid will pass through the point B, and it will be that curve along which a weight, by the force of its gravity, shall descend most swiftly from the point A to the point B. See* [**8**]

There is a legend that when Johann received the anonymous solution, he both chastened and admired it, placed the document down, and made the remark, "I recognize the lion by his paw," as seen in [**3**]. Whether or not this legend is true, it showcases Newton's abilities seen by his rivals in a humorous light.

Johann's solution to the brachistochrone takes a more creative path by incorporating Fermat's Principle of Least Time, which states that a ray of light will always take the path of shortest time, so light travels faster in materials with higher refractive index, and vice versa (see, e.g., [**12**]). Johann modeled his brachistochrone curve as a ray of light traveling through numerous layers of varying refractive indices, that becomes a continuous spectrum of varying refractive indices. After applying extensive calculus, the equations are composed of a differential equation, which Bernoulli recognized as the differential equation of a cycloid.

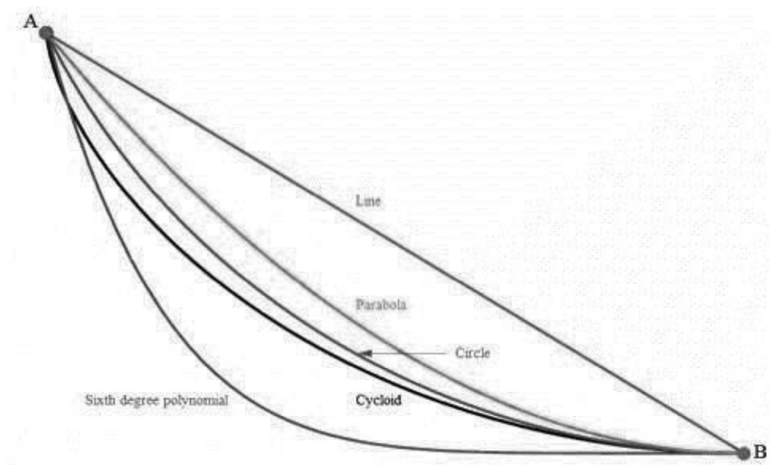


Image by MAA.

See [**12**]. Johann had figured out that the path of shortest time is indeed a cycloid. Johann ended his solution by making the statement, as seen in [**3**],

> *Before I end I must voice once more the admiration I feel for the unexpected identity of Huygens' tautochrone and my brachistochrone. I consider it especially remarkable that this coincidence can take place only under the hypothesis of Galileo, so that we even obtain from this a proof of its correctness. Nature always tends to act in the simplest way, and so it here lets one curve serve two different functions, while under any other hypothesis we should need two curves …*

Only five solutions were retrieved: those of Jakob Bernoulli, Leibniz, Newton, l'Hôpital, and, of course, Johann Bernoulli. Excluding l'Hôpital's solution, all were published in the 1697 publication of *Acta Eruditorum.* Johann praised these solutions by making the statement in [**8**]

> *...the three great nations, Germany, England, France, each one of their own to unite with myself in such a beautiful search, all finding the same truth.*

While it seems that Johann praised Jakob's solution, a bitter argument erupted between the brothers after the publication of *Acta Eruditorum.* Jakob, who did not want to be outdone by his younger brother, challenged Johann to Galileo's original question concerning the time to reach a vertical line rather than a point. Jakob posed the isoperimetric challenge to Johann in [**8**]

> *Given a starting point and a vertical line, of all the cycloids from the starting point with the same horizontal base, which will allow the point subjected only to uniform gravity, to reach the vertical line most quickly*

This would become a detestable episode for the brothers that would lead Johann to never step foot in Basel again until his brother's premature death from tuberculosis (see e.g., [**6**]). However, for other mathematicians, this would become a conflict of great value, as the problems argued between the brothers led to the founding of calculus variations (see e.g., [**8**]).

Johann married Dorothea Falkner in 1694. Their first child, Nicolaus (II) Bernoulli, was born in 1695 and was Johann's favorite child. His second child, born in 1697, only lived for six weeks. During this time in 1697, Johann became so severely ill that he was falsely reported to have died. He had two other children, Johann (II) and Daniel Bernoulli. All his children would become prominent mathematicians (see e.g., [**7**]).

Later, it seems as though Johann would have a relationship with his son, Daniel Bernoulli, that was similar to that of his brother, Jakob. Daniel would become the most distinguished of the second generation of Bernoulli as a mathematician. However, it is clear that his father would try to discourage him from mathematics, just as his own father did, insisting his son should be a merchant. Daniel would ironically end up following the same path his father did, studying medicine while his heart longed for mathematics, and would apply mathematical physics to medicine in order to gain his medical doctorate just like Johann had (see e.g., [**9**]). One of the examples of Johann's jealousy towards his son Daniel is seen in 1738, the year Daniel's reputation was first established as a fine mathematician when he published *Hydrodynamica*. In it, Daniel discussed the properties of basic importance in fluid flow, particularly pressure, density, and velocity, and set forth the fundamental relationship between them all. Then, later in 1738, Johann published *Hydraulica* as an attempt to take away his son's spotlight, yet another instance of Johann's antagonism toward his son.

Johann's bold, jealous, and pompous nature would be seen as a constant theme throughout his life, and not just toward his family and rivals. Johann could be best described by William Dunham given in [**1**]

> *(Johann) was a proud and arrogant man, as quick to demean the work of others as to praise that of himself. Second, any such praise was probably deserved.*

In one humorous instance in 1702, Johann was accused by one of his own students at the University of Groningen. Petrus Venhuysen published a pamphlet that accused Johann of supporting Descartes' philosophy and accusing him of opposing the Calvinist faith. In response, Johann wrote an extensive and humorous 12-page letter to the Governors of the University, which still exists today. It contains the following excerpt

> *... I would not have minded so much if [Venhuysen] had not been one of the worst students, an utter ignoramus, not known, respected, or believed by any man of learning, and he is certainly not in a position to blacken an honest man's name, let alone a professor known throughout the learned world... all my life I have professed my Reformed Christian belief, which I still do... he would have me pass for an unorthodox believer, a very heretic; indeed very wickedly he seeks to make me an abomination to the world, and to expose me to the vengeance of both the powers that be and the common people…*

See [**7**]. This demonstrates Johann's aggressive and understandably petty behavior, and would not be his only dispute while at Groningen, as scientists of the Cartesian persuasion held a strong distaste toward Johann introducing physics experiments in his teaching. Nevertheless, Johann did not let this discourage him from including physics experiments in his mathematics lectures.

Despite Johann's many strong quarrels, he was able to keep some relationships that were important to him, such as his friendship with Leibniz, whom Johann supported over Isaac Newton throughout the dispute over who discovered calculus first. This relationship would serve him well throughout his life as he was able to learn from Leibniz and apply what he learned from Leibniz to his own work.

Johann also had a meaningful correspondence with Leonhard Euler, having an important impact on the life of one of the greatest mathematicians in history. As mentioned before, Johann had lived with Euler's father, so it would make sense that Euler had known about him before entering the University of Basel. After earning his general education at the age of 14, Euler would convince Johann to give him private lessons, and through these lessons Johann would over eminent potential in Euler's abilities. In Euler's own unpublished autobiography, it is seen that this was true, as he wrote in [**10**]

> *... I soon found an opportunity to be introduced to a famous professor Johann Bernoulli. ... True, he was very busy and so refused flatly to give me private lessons; but he gave me much more valuable advice to start reading more difficult mathematical books on my own and to study them as diligently as I could; if I came across some obstacle or difficulty, I was given permission to visit him freely every Sunday afternoon and he kindly explained to me everything I could not understand ...*

In fact, it was Johann who convinced Euler's father to consent to Euler changing his study from theology to mathematics. Obviously, without Johann's friendship with Euler's father since they were undergraduates, this persuasion would not have been so easy. As such, it is important to note that not all Johann's friendships turned sour. Without Johann's encouragement, influence, and help, Euler's mathematical journey would have been more difficult to achieve so quickly and his greatest work would have been longer delayed had he continued studying theology under his father's wishes. During Euler's time in Basel, he reconstructed numerous pieces of work that he had read with the help of Johann, such as those of Varignon, Descartes, Newton, Galileo, van Schooten, Jacob Bernoulli, Hermann, Taylor, and Wallis (see e.g., [**10**]). Euler would hold close ties with the Bernoulli family, and they would play an important role throughout the rest of his life.

Johann made several important contributions to mechanics with work on kinetic energy, which had been a topic that mathematicians had argued over before Johann's time. Throughout Johann's life, he obtained prominent fame through his work. He was elected a fellow of the academies of Paris, Berlin, London, St. Petersburg, and Bologna. Johann died at the age of eighty on January 1st, 1748. Johann was such an influential and phenomenal mathematician that "Archimedes of his age" was engraved on his tombstone (see e.g., [**7**]).

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